< 정답 및 해설 >

<정답>

문항	1	2	3	4	5	6	7	8	9	10	11	12	13
정답	②	③	①	④	④	②	③	②	④	②	①	①	④
문항	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
정답	③	②	①	②	③	④	②	③	①	①	④	③	

<해설>

1. $g'(y)=f(y),\ g''(y)=f'(y),\ f'(x)=2x\sqrt{1+3x^2}$ 이므로 $g''(1)=f'(1)=2\sqrt{1+3}=4$

2. $f'(x)=\cos^{-1}x-\dfrac{x}{\sqrt{1-x^2}}+\dfrac{x}{\sqrt{1-x^2}}=\cos^{-1}x$ 이므로 $f'\left(\dfrac{1}{2}\right)=\cos^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{3}$

3. 극좌표 $\left(4,\ \dfrac{\pi}{3}\right)$를 직교좌표로 변환하면 $\left(4\cos\dfrac{\pi}{3},\ 4\sin\dfrac{\pi}{3}\right)=(2,\ 2\sqrt{3})$

   극좌표 $\left(6,\ \dfrac{2\pi}{3}\right)$를 직교좌표로 변환하면 $\left(6\cos\dfrac{2\pi}{3},\ 6\sin\dfrac{2\pi}{3}\right)=(-3,\ 3\sqrt{3})$

   따라서 두 점 사이의 거리를 d라 하면

   $$d=\sqrt{(2-(-3))^2+(2\sqrt{3}-3\sqrt{3})^2}=\sqrt{25+3}=\sqrt{28}=2\sqrt{7}$$

4. $\nabla f(x,y,z)=<2xy,\ x^2+2yz,\ y^2>,\ \nabla f(1,\ 2,\ 3)=<4,\ 13,\ 4>$

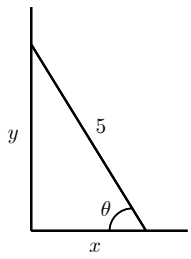
   $|v|=\sqrt{2^2+(-1)^2+2^2}=3,\ u=\dfrac{v}{|v|}=\left\langle\dfrac{2}{3},\ -\dfrac{1}{3},\ \dfrac{2}{3}\right\rangle$

   $$D_u f(1,\ 2,\ 3)=<4,\ 13,\ 4>\ \cdot\left\langle\frac{2}{3},\ -\frac{1}{3},\ \frac{2}{3}\right\rangle=1$$

5. $f_{zxy}(x,y,z)=f_{yxz}(x,y,z)$ 이므로 $f(x,y,z)=x^2y^3z^4+\tan^{-1}\left(x^2\sqrt{z}\right)$에서

   $$f_y(x,y,z)=3x^2y^2z^4,\ f_{yx}(x,y,z)=6xy^2z^4,\ f_{yxz}(x,y,z)=24xy^2z^3$$

6. 벽면과 사다리 바닥 사이의 거리를 x, 지면과 사다리 꼭대기 사이의 거리를 y라 하면, $\frac{dx}{dt}=1$, $x=3$일 때 $y=4$이므로 $\sin\theta=\frac{4}{5}$



$\cos\theta=\frac{x}{5}$ 에서 $-\sin\theta\,\frac{d\theta}{dt}=\frac{1}{5}\frac{dx}{dt}$

$$\frac{d\theta}{dt}=-\frac{1}{5\sin\theta}\,\frac{dx}{dt}=-\frac{1}{5\times 4/5}\,(1)=-\frac{1}{4}\,(\text{rad/s})$$

7. $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{3\sin\theta}{3(1-\cos\theta)}=\frac{\sin\theta}{1-\cos\theta}$ 이므로 $\theta=\pi/6$일 때 기울기는

$$\frac{dy}{dx}=\frac{1/2}{1-\sqrt{3}/2}=\frac{1}{2-\sqrt{3}}=2+\sqrt{3}$$

8. $\frac{dy}{dx}=\sinh 2x$, 곡선의 길이를 L이라 하면

$$L=\int_0^1\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$=\int_0^1\sqrt{1+\sinh^2 2x}\,dx$$

$$=\int_0^1\sqrt{\cosh^2 2x}\,dx=\int_0^1\cosh 2x\,dx=\left[\frac{1}{2}\sinh 2x\right]_0^1=\frac{1}{2}\sinh 2$$

9. $V=\int_1^e\pi y^2dx=\pi\int_1^e\ln x\,dx=\pi[x\ln x-x]_1^e=\pi$

10. $r(t)=<\ln t, 2t, t^2>$, $r'(t)=<1/t, 2, 2t>$

점 $(0, 2, 1)$은 $t=1$에 대응한다. $r'(1)=<1, 2, 2>$는 법평면의 법선벡터이므로

법평면의 방정식은 $1(x-0)+2(y-2)+2(z-1)=0$, $x+2y+2z=6$이므로

$a=1, b=2, c=2$, $a+b+c=5$

11. 점 $(1, 1, 2)$는 $t=1$에 대응한다.

$r'(t)=<1, 1, 2t>$, $r'(1)=<1, 1, 2>$, $|r'(1)|=\sqrt{6}$

$r''(t)=<0, 0, 2>$, $r''(1)=<0, 0, 2>$

$r'(1)\times r''(1)=<2, -2, 0>$, $|r'(1)\times r''(1)|=2\sqrt{2}$

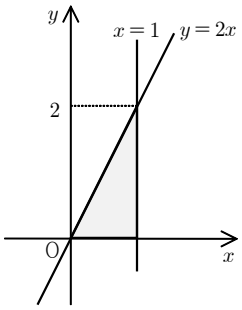
$$\kappa(1)=\frac{|r'(1)\times r''(1)|}{|r'(1)|^3}=\frac{2\sqrt{2}}{6\sqrt{6}}=\frac{\sqrt{3}}{9}$$

12. $\displaystyle\int_0^2\int_{y/2}^1 y\cos(x^3-1)\,dx\,dy=\int_0^1\int_0^{2x} y\cos(x^3-1)\,dy\,dx$

$$=\int_0^1\cos(x^3-1)\left[\frac{1}{2}y^2\right]_0^{2x}dx$$

$$=\int_0^1 2x^2\cos(x^3-1)dx$$

$$=\left[\frac{2}{3}\sin(x^3-1)\right]_0^1=\frac{2}{3}\sin 1$$



13. 영역 $D=\{(x,y)\mid 0\le x\le 2,\ 0\le y\le \sqrt{4-x^2}\}$는 극좌표로 나타내면

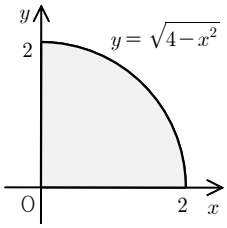
$\left\{(r,\theta)\mid 0\le r\le 2,\ 0\le\theta\le\dfrac{\pi}{2}\right\}$이므로

$$\iint_D e^{-x^2-y^2}dA=\int_0^{\pi/2}\int_0^2 e^{-r^2}r\,dr\,d\theta$$

$$=\int_0^{\pi/2}1\,d\theta\int_0^2 r\,e^{-r^2}dr$$

$$=\frac{\pi}{2}\left(-\frac{1}{2}\left[e^{-r^2}\right]_0^2\right)$$

$$=\frac{\pi}{4}(1-e^{-4})$$



14. $M=kx^2y+e^y-2x$, $N=xe^y+x^3+2y$라 하면

$M_y=kx^2+e^y$, $N_x=e^y+3x^2$이므로 $M_y=N_x$에서 $k=3$

15. $(x^2+1)dy=\dfrac{x}{\sin y}dx$, $\sin y\,dy=\dfrac{x}{x^2+1}dx$, $-\cos y=\dfrac{1}{2}\ln(x^2+1)+c$

$y(0)=\pi$에서 $c=1$이므로 $\cos y+\dfrac{1}{2}\ln(x^2+1)+1=0$

16. 보조방정식의 근은 $m_1=\dfrac{1}{2}+i$, $m_2=\dfrac{1}{2}-i$이므로 보조방정식은

$$\left[m-\left(\frac{1}{2}+i\right)\right]\left[m-\left(\frac{1}{2}-i\right)\right]=0,\ m^2-m+\frac{5}{4}=0,\ \frac{4}{5}m^2-\frac{4}{5}m+1=0$$

$\Leftrightarrow am(m-1)+bm+1=0$, $am^2+(b-a)m+1=0$에서 $a=\dfrac{4}{5}$, $b=0$이므로

$$a+b=\frac{4}{5}$$

17. $\frac{dy}{dx}=1+(y-x)^2$ 에서 $w=y-x$ 라 하면

$$w'=y'-1=1+w^2-1,\ w'=w^2,\ \frac{1}{w^2}dw=dx,\ -\frac{1}{w}=x+c$$

에서 $y=x-\frac{1}{x+c}$. $y(0)=\frac{1}{2}$ 이므로 $c=-2$,

따라서 $y(x)=x-\frac{1}{x-2}$ 이고 $y(3)=2$

18. $y''-\frac{1}{x-1}y'+\frac{1}{x^2-x}y=0$ 이고

$$u(x)=\int\frac{1}{x^2}\ e^{-\int\frac{-1}{x-1}dx}\,dx=\int\frac{1}{x^2}(x-1)dx=\int\left(\frac{1}{x}-\frac{1}{x^2}\right)dx=\ln x+\frac{1}{x}$$

따라서 $y_2(x)=u(x)y_1(x)=x\ln x+1$

19. $t=\ln\frac{1}{x}$ 이라 하면 $x=e^{-t}$, $dx=-e^{-t}dt$

$$\int_0^1 x^2\left(\ln\frac{1}{x}\right)^{\frac{1}{2}}dx=\int_\infty^0 e^{-2t}\,t^{\frac{1}{2}}(-e^{-t}dt)=\int_0^\infty t^{\frac{1}{2}}e^{-3t}dt$$

에서 $u=3t$ 라 하면 $t=\frac{u}{3}$, $dt=\frac{1}{3}du$ 이므로

$$\int_0^\infty t^{\frac{1}{2}}e^{-3t}dt=\frac{1}{3}\int_0^\infty\left(\frac{u}{3}\right)^{\frac{1}{2}}e^{-u}\,du$$

$$=\frac{1}{3\sqrt{3}}\int_0^\infty u^{\frac{1}{2}}e^{-u}\,du$$

$$=\frac{1}{3\sqrt{3}}\Gamma\left(\frac{3}{2}\right)$$

$$=\frac{1}{3\sqrt{3}}\times\frac{1}{2}\times\Gamma\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{18}\times\sqrt{\pi}=\frac{\sqrt{3\pi}}{18}$$

20. $f(t)=\sin t-\sin t\,u(t-\pi)=\sin t+\sin(t-\pi)\,u(t-\pi)$

$\mathcal{L}\{f(t)\}=\mathcal{L}\{\sin t\}+\mathcal{L}\{\sin(t-\pi)\,u(t-\pi)\}$

$$\mathcal{L}\{f(t)\}=\frac{1}{s^2+1}+\left(\frac{1}{s^2+1}\right)e^{-\pi s}=\frac{1}{s^2+1}(1+e^{-\pi s})$$

21. $y''+y=0$의 보조방정식은 $\lambda^2+1=0$에서 $\lambda=-i,\ i$이므로 $y''+y=0$의 일차독립인 해는 $y_1=\cos x,\ y_2=\sin x$. 따라서 $y_c=c_1\cos x+c_2\sin x$

$$W=\begin{vmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{vmatrix}=1,\quad W_1=\begin{vmatrix}0 & \sin x\\ \csc x & \cos x\end{vmatrix}=-1,$$

$$W_2=\begin{vmatrix}\cos x & 0\\ -\sin x & \csc x\end{vmatrix}=\cos x\csc x=\frac{\cos x}{\sin x}$$

$u_1'=\dfrac{W_1}{W}=-1$에서 $u_1=-x,\ u_2'=\dfrac{W_2}{W}=\dfrac{\cos x}{\sin x}$에서 $u_2=\ln|\sin x|$

이므로 $y''+y=\cos x$의 특수해 y_p는

$$y_p=u_1y_1+u_2y_2=-x\cos x+\sin x\ln|\sin x|$$

일반해는 $y=y_c+y_p=c_1\cos x+c_2\sin x-x\cos x+\sin x\ln|\sin x|$

22. $f(t)=t^2*\cos t$의 라플라스 변환 $F(s)$는

$$F(s)=\mathcal{L}\{t^2*\cos t\}=\frac{2}{s^3}\times\frac{s}{s^2+1}=\frac{2}{s^2(s^2+1)}$$

$$f(t)=\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{\frac{2}{s^2(s^2+1)}\right\}=2\mathcal{L}^{-1}\left\{\frac{1}{s^2}-\frac{1}{s^2+1}\right\}$$

이므로 $f(t)=2(t-\sin t)$. 따라서 $f\left(\dfrac{\pi}{2}\right)=2\left(\dfrac{\pi}{2}-1\right)=\pi-2$

23. $F'(s)=-\mathcal{L}\{tf(t)\}$이므로 $\mathcal{L}^{-1}\{F'(s)\}=-tf(t)$

$$f(t)=-\frac{1}{t}\mathcal{L}^{-1}\{F'(s)\}=-\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{d}{ds}[\ln(s-2)-\ln(s+3)]\right\}$$

$$=-\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{1}{s-2}-\frac{1}{s+3}\right\}=-\frac{1}{t}\left(e^{2t}-e^{-3t}\right)$$

$$f(1)=e^{-3}-e^2$$

24. $y(t)=\cos t+e^{-t}\displaystyle\int_0^t e^u y(u)\,du$

$$=\cos t+\int_0^t e^{-(t-u)}\,y(u)\,du=\cos t+e^{-t}*y(t)$$

$\mathcal{L}\{y(t)\}=Y(s)$라 하고 라플라스 변환을 취하면

$$\mathcal{L}\{y(t)\}=\mathcal{L}\{\cos t\}+\mathcal{L}\{e^{-t}*y(t)\}$$

$$Y(s)=\frac{s}{s^2+1}+\frac{1}{s+1}Y(s),\quad \frac{s}{s+1}Y(s)=\frac{s}{s^2+1}$$

에서 $Y(s)=\dfrac{s+1}{s^2+1}=\dfrac{s}{s^2+1}+\dfrac{1}{s^2+1}$이므로 $y(t)=\cos t+\sin t$

25. $\mathcal{L}\{x(t)\}=X(s)$, $\mathcal{L}\{y(t)\}=Y(s)$라 하고 라플라스 변환을 취하면

$$\begin{cases} sX(s)-x(0)=X(s)-2Y(s) \\ sY(s)-y(0)=5X(s)-Y(s) \end{cases}$$

이고 $x(0)=1$, $y(0)=2$이므로

$$\begin{cases} sX(s)-1=X(s)-2Y(s) \\ sY(s)-2=5X(s)-Y(s) \end{cases} \Leftrightarrow \begin{cases} (s-1)X(s)+2Y(s)=1 \\ 5X(s)-(s+1)Y(s)=-2 \end{cases}$$

에서

$$X(s)=\frac{s-3}{s^2+9}=\frac{s}{s^2+9}-\frac{3}{s^2+9}$$

따라서 $x(t)=\mathcal{L}^{-1}\{X(s)\}=\cos 3t-\sin 3t$이므로 $x\left(\frac{\pi}{3}\right)=-1$